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LEADING EDGE SEPARATION CRITERION FOR AN OSCILLATING AIRFOIL

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ABSTRACT

A Unsteady flow about the well-rounded nose of a subsonic airfoil is investigated from the viewpoint of leading edge separation. For an airfoil undergoing forced pitching and heaving motions in a uniform flow, ~~the~~ fluid accelerations about the leading edge can be enormous according to inviscid flow theory. Such accelerations are limited by viscous flow and separation realities.

The method of matched asymptotic expansions is used to develop a uniformly valid first order approximation to the inviscid flow about the ~~airfoil. The~~ inviscid flow ~~about the~~ airfoil's which leading edge is driven by a history-dependent term related to the airfoil's transverse motions. Applying this flow to the laminar boundary layer flow at the airfoil nose produces possibilities for a laminar boundary layer to separate. A method is

The Moore-Rott-Sears (M-R-S) conditions for unsteady boundary layer separation do not appear to be useful for this problem. A methodology is proposed for predicting leading edge dynamic stall based upon relating properties of the envelope of the unsteady part of the boundary layer speed and shear stress to the steady (D.C.) part of the boundary layer flow. The development is proposed as a tool for determining the useful limit for applying attached inviscid airfoil flow theory.

NOMENCLATURE

a	complex amplitude
$\underline{a}$	acceleration vector
$\bar{a}_0(t)$	suction strength
$b_0(t)$	Fourier velocity coefficient
c	constant related to on-set flow
$F(\sigma)$	$= \mathrm{Re}\ \Theta$, Theodorsen
$G(\sigma)$	$= \mathrm{Im}\ \Theta$, Theodorsen
f	complex velocity potential
$g(t)$	Bernoulli constant
$h(x,t)$	displacement function
i	unit complex number, space
j	unit complex number, time
$k(\eta)$	steady viscous function
$K_0(j\sigma)$	modified Bessel function
p	local pressure
q	flow speed
R	Reynolds number based on chord
r	nose radius of curvature
t	time
U, U_s	on-set speed
U_t	tangential speed
(u,v)	velocity components
(u_1,u_2)	velocity components
(x,y)	coordinate pair
(X,Y)	coordinate pair
Z	$= X + iY$, complex position
(β,α)	coordinate pair
ζ	$= \zeta_R + i\zeta_I$, complex flow function
η	$= \sqrt{R}\ y$, similarity parameter

θ	$= \omega t$
$\Theta(\sigma)$	Theodorsen function
κ	$= \beta + i\alpha$, complex variable
ξ_0	heaving parameter
ξ_1, ξ_2	pitching parameters
ρ	fluid density
σ	reduced frequency of the motion
Φ	acceleration potential
ϕ	velocity potential
ψ	stream function
ν	kinematic viscosity
ω	circular frequency

INTRODUCTION

Based upon the linear aerodynamic theory, Wu[1] developed hydrodynamic analyses for optimum pitching-heaving motion of a rigid wing. The optimum problem was to minimize the time averaged energy loss coefficient C_E under the constraint that the time averaged thrust coefficient C_T was fixed. That is, for $C_T = \overline{C}_{T_0} > 0$, what phasing between the pitching-heaving motions minimizes the rate at which energy is lost to the flow in the shedding of kinetic energy at the airfoil's trailing edge? The time averaged thrust coefficient is comprised of a mean thrust delivered from the plate surface and a mean suction due to the inviscid flow accelerating about the airfoil leading edge. Wu determined that the ratio of mean suction thrust coefficient to total thrust coefficient C_S/C_T has a minimum at a reduced frequency of the motion $\sigma = \sigma_m(\overline{C}_{T_0})$. Outside of the region of $\sigma = \sigma_m$, the suction force can become so large that leading edge stall is inevitable. The present paper advances a methodology for determining the limit on the attainable C_S for oscillatory motion of a thin airfoil based upon the behavior of a laminar boundary layer near the airfoil's leading edge stagnation point.

Sychev's impressive manuscript[2] shows that under certain restrictions on the acceleration of the flow, the point of separation is inviscid in nature. This result contrasts with the steady separation problem. Furthermore, the unsteady separation point is not situated on the surface of the body.

The approach taken herein has been to treat the leading edge separation problem in the context of its relationship to the full flow about the oscillating airfoil. In treating the problem in this way, attention is given to the situation that leading edge separation appears to be a local phenomenon in the sense that it occurs in a region where the airfoil has large curvature, and experiences a stagnation point flow. Once dynamic separation has occurred, the aerodynamic theory used to describe the attached flow about the airfoil no longer applies.

The approach taken connects the linear airfoil theory and the unsteady oscillating laminar boundary layer theory. The latter has been developed, for example, in References [3] - [7].

It is well known that the linear airfoil theory breaks down at the airfoil leading edge. This deficiency has been eliminated in this development by asymptotic matching with a local inviscid unsteady solution about an osculating parabola. Results from matching then open the way for investigating the behavior of the laminar boundary layer in the region of the airfoil nose. This boundary layer is driven by the gross oscillatory motion of the airfoil and by its steady forward speed. The unsteady driver turns out to be directly related to the suction strength about the airfoil's leading edge, (as derived from the inviscid theory).

Flat plate unsteady boundary layer theory is applicable in this investigation whenever the boundary layer thickness is small compared with the radius of curvature of the airfoil's nose. We tacitly assume this to be the case.

KINEMATICS

We consider the small amplitude heaving and pitching motion of a thin symmetric airfoil in a steady uniform stream $\underline{U}$. To describe the kinematics of such motion we introduce a Cartesian coordinate system (x,y) with the x-axis aligned with the airfoil's mean chord line. The direction of the free stream is along the positive x-axis. The airfoil's transverse displacements then occur along the y-axis and are prescribed by a function h(x,t) of chordwise position x and time t. Figure 1 illustrates the transverse displacement of a typical wing section with respect to the (x,y) coordinate system.

The inviscid wing boundary condition requires the normal velocity of the wing relative to the (x,y) coordinate system be equal to the normal velocity $\underline{u} \cdot \underline{n}$ of the fluid adjacent to the wing. Here $\underline{n}$ is the unit outward normal vector on the wing and $\underline{u}$ is the fluid velocity relative to the inertial reference frame resolved into components (U+u,v) along the (x,y) axes.

Neglecting products of small quantities compared with those occurring linearly, the kinematic boundary condition specifies the v-component of the fluid velocity adjacent to the wing. Consistent with approximations already made, this component can be specified along the x-axis, giving

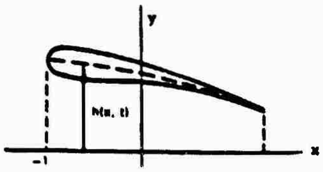

Fig.1- Airfoil Displacement

$$v(x,t) = \left[\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right] h(x,t); \quad \begin{cases} y = 0^{\pm} \\ |x| < 1 \\ t > 0 \end{cases}$$

For heaving and pitching motion of a rigid plate at the frequency ω, the displacement function h(x,t) can be expressed as

$$h(x,t) = [\xi_0/2 + (\xi_1 + j\xi_2)] \exp (j\omega t)$$

where $|x| < 1$ and ξ_0, ξ_1, and ξ_2 are real. The pitching axis is at the midchord, x = 0. The amplitudes of pitching and heaving are $|\xi_1 + j\xi_2|$ and $\xi_0/2$, respectively. Pitching leads the heaving motion by a phase angle $\tan^{-1}(\xi_2/\xi_1)$.

A convenient form of expressing v(x,t) is by its Fourier cosine series. This series contains only two two terms for the specified transverse motion. That is,

$$v(x,t) = b_0(t)/2 + b_1(t) \cos \theta \qquad (1)$$

$$b_0(t) \equiv U[2\xi_1 + j(2\xi_1 + \sigma\xi_0)] e^{j\omega t} \qquad (2)$$

$$b_1(t) \equiv -U\sigma(\xi_2 - j\xi_1) e^{j\omega t} \qquad (3)$$

where $x = \cos \theta$, $\theta = \omega t$ and $\sigma = \omega/U$ is the reduced frequency of the motion based on the unit half-chord.

INVISCID DYNAMICS

In an incompressible flow field devoid of external forces and internal viscosity, the principle of conservation of mass leads to the expression $\underline{\nabla} \cdot \underline{u} = 0$. Conservation of rectilinear momentum leads to the Euler equation wherein the pressure gradient is balanced by the fluid acceleration $\underline{a} = -\rho^{-1} \underline{\nabla} p$. This equation is valid in any inertial reference frame. The absolute acceleration $\underline{a}$ measures the rate of change of $\underline{u}$ following a particle, ρ is the fluid density, and p is the local instantaneous pressure. If in addition, the flow field is irrotational then $\underline{\nabla} \times \underline{u} = 0$ and a scalar function ϕ exists such that $\underline{u} = \underline{U} + \underline{\nabla}\phi$ and $\nabla^2 \phi = 0$. Here the flow field is defined to be the region exterior to the wing and its shed vortex sheet.

An integral of the momentum equation can be obtained by substituting $\underline{u} = \underline{U} + \underline{\nabla}\phi$ into the acceleration $\underline{a}$ and neglecting products of small quantities. The resulting integral becomes

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\phi(x,y,t) = \Phi(x,y,t)$$

where Φ, the Prandtl acceleration potential, measures the variation of the pressure from the static level, $\Phi(x,y,t) = [p_\infty - p(x,y,t)]/\rho$. Applying the Laplace operator ∇^2 to the above integral results in $\nabla^2 \Phi = 0$ as the relevant field equation for the unsteady inviscid incompressible thin airfoil problem.

INVISCID OUTER PROBLEM

The appropriate airfoil boundary value problem has been solved in terms of the acceleration potential by Wu[8] in an elegant treatise on the hydrodynamics of swimming propulsion. In particular, he has determined that the pressure difference across the wing, $|x| < 1$ is

$$\Delta p = p^-(x,t) - p^+(x,t) = 2\rho\Phi^+(x,t) \qquad (4)$$

where $\Phi^+(x,t)$ is the acceleration potential evaluated along the x-axis, $y = 0^+$.

For time harmonic motion that has persisted indefinitely, the acceleration potential evaluated on the topside of the wing is

$$\Phi^+(x,t) = \frac{1}{2}Ua_0(t)\sqrt{\frac{1-x}{1+x}} + \Psi_1(x,t) \qquad (5)$$

where

$$\Psi_1(x,t) = \frac{1}{\pi}\int_{-1}^{1}\sqrt{\frac{1-x^2}{1-\xi^2}}\,\frac{\Lambda(\xi,t)}{(\xi-x)}\,d\xi \qquad (6)$$

$$\Lambda(\xi,t) = -\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial \xi}\right)\int_{-1}^{\xi} v(x,t)\,dx \qquad (7)$$

$$a_0(t) = b_1(t) - \Theta(\sigma)[b_0(t) + b_1(t)] \qquad (8)$$

In this last expression, $\Theta(\sigma)$ is the Theodorsen function, which is expressible in terms of modified K-type Bessel functions, as

$$\Theta(\sigma) = K_1(j\sigma)/[K_0(j\sigma) + K_1(j\sigma)]$$
$$\Theta(\sigma) = F(\sigma) + jG(\sigma) \qquad (9)$$

Substituting Eq. (1) into Eq. (7) gives

$$\Lambda(\xi,t) = c_0(t) + \xi c_1(t) + \xi^2 c_2(t) \qquad (10)$$

where

$$\left.\begin{aligned} c_0(t) &= -[\dot{b}_0 - \dot{b}_1 + Ub_0]/2 \\ c_1(t) &= -[Ub_1 + \dot{b}_0/2] \\ c_2(t) &= -\dot{b}_1/2 \end{aligned}\right\} \qquad (11)$$

Here, the dot denotes time differentation. Substituting Eqs. (5), (6), (10) and (11) into Eq. (4), gives the pressure jump across the wing, $|x| < 1$. That is,

$$\frac{\Delta p}{\rho} = Ua_0\sqrt{\frac{1-x}{1+x}} + 2(c_1 + xc_2)\sqrt{1-x^2} \qquad (12)$$

We notice that this pressure jump expression has a square root singularity at the leading edge of the airfoil. The quantity, however, is integrable over the wing chord and is used quite effectively to yield quantitative estimates of global quantities such as sectional lift, moment, thrust, power input necessary to sustain the motion, energy loss due to vortex shedding at the trailing edge, etc. However, the result is useless as a means for providing flow detail in the vicinity of the leading edge. The reason for this of course is clear. In the neighborhood of the leading edge there occurs a stagnation point. As a consequence, the perturbation does not remain small compared with the on-set flow as is required by the linear theory. The linear theory is therefore not uniformly valid and breaks down in the region surrounding the airfoil nose. Our first objective is to correct this deficiency of the linear airfoil theory by determining the appropriate correction for the construction of a uniformly valid first order solution to the inviscid unsteady airfoil problem.

INVISCID INNER PROBLEM

The simplest representation of the inviscid flow about a well-rounded nose of a thin airfoil that preserves the essential character of the problem is the flow about an infinite parabola. Upon magnifying the detail at the leading edge what appears is the flow about a so-called osculating parabola. Such a flow is comprised of an on-set stagnation point flow and a tangential or parallell flow. Figure 2 illustrates the inviscid flow about a parabolic cylinder. The geometry is described by a coordinate system (X,Y) with origin at the base of the parabola and with X being the axis of the parabola.

If we employ the conformal transformation

$$Z = \kappa - \kappa^2 \qquad (13)$$

where

$$Z = X + iY \qquad (14)$$

$$\kappa = \beta + i\alpha \qquad (15)$$

the flow field in the physical Z-plane is tranformed onto the left half κ-plane, as presented in Figure 3. That is, the conformal transformation takes the parabola onto a straight line. The upper branch of the parabola goes to the positive imaginary κ-axis. The lower branch goes to the negative κ-axis.

Equation (13) can be used to provide the inverse transformation yielding κ as a function of Z. By selecting the negative branch so that

$$\kappa = [1 - \sqrt{1 - 4Z}\,]/2 \qquad (16)$$

then the parabola $\beta = 0$ yields $\alpha = \pm\sqrt{X} = Y$.

In terms of the complex velocity potential function f, the stagnation point flow and the parallel flow can be readily represented in the κ-plane by

$$f = -U_s\kappa^2 - iU_p\kappa = \phi + i\psi \qquad (17)$$

where U_s and U_p are quantitites to be determined and ϕ,ψ are the velocity potential function and stream function, respectively.

Substituting Eq. (16) into Eq. (17) yields

$$f = (U_s + iU_p)[\sqrt{1-4Z} - 1]/2 + U_sZ \qquad (18)$$

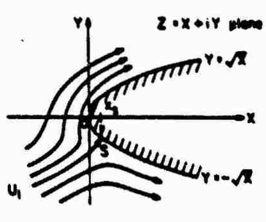

Fig 2-Airfoil LE Flow

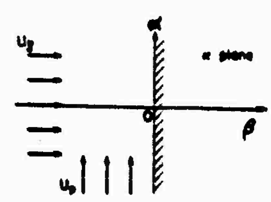

Fig.3-Mapped LE Flow

From the above expression we can calculate the nonlinear pressure jump across the nose of the airfoil. This we do by using the Bernoulli equation for unsteady incompressible flow

$$\frac{p}{\rho} + \frac{1}{2}(q^2) + \frac{\partial\phi}{\partial t} = g(t)$$

Across the parabolic airfoil nose the pressure jump is $\Delta p = p^- - p^+$. Therefore,

$$\Delta p/\rho = -\Delta\phi_t - \Delta(q^2/2)$$

The local time variation of the potential function ϕ_t can be obtained directly from the real part of Eq. (18). This equation can also be used to obtain q since $q = |df/dZ|$. Consequently, Δp across the nose of the parabola $Y = \pm\sqrt{X}$ is

$$\Delta p/\rho = 2\dot{U}_p\sqrt{X} - 4U_sU_p\sqrt{X}/(1+4X) \qquad (19)$$

This expression is the leading edge counterpart to Eq. (12). Notice that it does not break down at the leading edge.

MATCHING

To obtain a uniformly valid first order approximation to the pressure jump across the wing, Eqs. (12) and (19) should be matched in some overlapping region where both are presumed valid. In Eq. (12), if we make the substitution $\xi = 1 + x$ and take the limit as ξ tends to zero, then

$$\Delta p/\rho \sim \sqrt{2/\xi}\, Ua_0 + 2[(c_1-c_2) - Ua_0/4]\sqrt{2\xi} \qquad (20)$$

In Eq. (19), set $X = \xi$ and let ξ tend to infinity. This limiting process results in

$$\Delta p/\rho \sim -U_sU_p/\sqrt{\xi} + 2\dot{U}_p\sqrt{\xi} \qquad (21)$$

Comparing coefficients of the $\xi^{-1/2}$ term gives

$$U_s = U \qquad (22)$$

$$\left.\begin{aligned} U_p(t) &= -\sqrt{2}\, a_0(t) \\ U_p(t) &= \sqrt{2}\, U(\lambda+i\mu)e^{j\omega t} \end{aligned}\right\} \qquad (23)$$

where

$$\left.\begin{aligned} \lambda &= \sigma\xi_2 + 2F\sigma_1 - G\sigma_2 \\ \mu &= -\sigma\xi_1 + 2G\sigma_1 + F\sigma_2 \\ \sigma_1 &= \xi_1 - \sigma\xi_2/2 \\ \sigma_2 &= \sigma\xi_1 + 2\xi_2 + \sigma\xi_0 \end{aligned}\right\} \qquad (24)$$

Consequently, to leading order, the magnitude of the stagnation point flow is equal to the uniform on-set flow U. This we expected on the basis of the steady flow analog to this problem. See Van Dyke's book[9], §4.9. An interesting result is that the parallel flow about the airfoil leading edge is directly related to the strength of the leading edge suction $a_0(t)$. This term is the only quantity that contains the history of the motion. Such motion history is due to vortex shedding at the airfoil's trailing edge. Thus to leading order, the flow about the leading edge is driven by the dynamics and kinematics of vortex shedding at the trailing edge.

To construct a uniformly valid first approximation for the pressure jump across the airfoil surface we add the inner solution (19) to the outer solution (12) and subtract the common part. The result is

$$\frac{\Delta p}{\rho} = Ua_0\left[\frac{\sqrt{1-x} - \sqrt{2}}{\sqrt{1+x}}\right] + \sqrt{1+x}\, S(x,t) \qquad (25)$$

where

$$S(x,t) = 2(c_1+xc_2)\sqrt{1-x} - \sqrt{2}\left[\dot{a}_0 - \frac{4Ua_0}{\sqrt{5+4x}}\right]$$

Notice that there is no singularity in the above expression in the range $|x| < 1$ where Δp is evaluated.

LAMINAR BOUNDARY LAYER PROBLEM

As a result of the asymptotic matching technique employed for this problem, we have determined that the flow about the airfoil's leading edge oscillates in direct proportion to the strength of the leading edge suction. The actual flow at the edge of the boundary layer about the airfoil nose can be estimated from Eq. (17) by taking the derivative $df/d\kappa = u_1 - iu_2$. Here u_1 and u_2 are the velocity components along the (β,α) axes of κ. We obtain, for $\beta = 0$, the local flow along the parabolic surface. That is,

$$u_1 = 0$$

$$u_2 = U_p + 2\alpha U_s \qquad (26)$$

Therefore, as one moves along the parabola (either positively or negatively away from α equal zero) the mean speed increases in magnitude.

When the on-set stream does not oscillate but the surface oscillates, the situation differs from the oscillating dividing streamline case only by the superposition of a uniform, though non-constant transverse velocity which has no effect on the relative motion (cf. Ref. 6). Taking advantage of these facts, the relevant boundary layer problem to consider is that of a two-dimensional flow against an infinite flat plate normal to the free stream where the plate makes transverse oscillations in its own plane. This is a classical problem in boundary layer theory[6] whose exact solution depends on a set of ordinary differential equations containing the reduced frequency σ as a parameter. To conform with standard notation for this problem, we reuse some notation already used for another purpose. In as much as the principal results of the analyses thus far are embodied in Eqs. (22) - (24) which are independent of coordinate system -- no confusion will arise.

We now introduce a Cartesian coordinate system (x,y) with the x-axis along a flat plate and the y-axis normal to it so that $x = 0$ is the dividing streamline in the steady flow outside the boundary layer on the plate. Let (u,v) be the corresponding velocity components. Outside the boundary layer suppose

$$u = cx \quad \text{as} \quad y \to \infty \tag{27}$$

Let the plate oscillate along the x-axis so that

$$u = ae^{j\omega t} \quad \text{at} \quad y = 0 \tag{28}$$

where c and ω are real constants. The amplitude of the plate's speed 'a' is here a complex constant. It is understood that the real part is to be taken for all physical quantities.

Comparing Eqs. (22), (23), (26)-(28) we have

$$\left.\begin{aligned} c &= 2U \\ a &= a_R + ia_I = \sqrt{2}\,U(\lambda + i\mu) \end{aligned}\right\} \tag{29}$$

The boundary layer equations are

$$u_t + uu_x + vu_y = c^2x + \nu u_{yy}$$

$$u_x + v_y = 0$$

where ν is the kinematic viscosity. These equations are to be solved with

$$u = ae^{j\omega t}, \; v = 0; \quad y = 0$$

$$u = cx\,; \quad y \to \infty$$

A similarity solution is known to satisfy the problem. The solution form is

$$u = 2Uxk'(\eta) + ae^{j\omega t}\zeta(\eta) \tag{30}$$

$$v = -(2U\nu)^{1/2}k(\eta)$$

$$\zeta(\eta) = \zeta_R(\eta) + i\zeta_I(\eta)$$

$$\eta = \sqrt{R}\,y$$

$R = 2U/\nu$; Reynolds number based on chord length

where $k(\eta)$, $\zeta_R(\eta)$, $\zeta_I(\eta)$ satisfy ordinary differential equations. That is,

$$\left.\begin{aligned} &k''' + kk'' + k'k' + 1 = 0 \\ &k(0) + k'(0) = 0, \; k'(\infty) = 1 \end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned} &\zeta_R'' + k\zeta_R' - k'\zeta_R + \sigma\zeta_I/2 = 0 \\ &\zeta_I'' + k\zeta_I' - k'\zeta_I - \sigma\zeta_R/2 = 0 \\ &\zeta_R(0) = 1, \; \zeta_I(0) = \zeta_R(\infty) = \zeta_I(\infty) = 0 \end{aligned}\right\} \tag{32}$$

The nonlinear k-problem is the classical Hiemenz stagnation point flow. Note that the linear ζ-problem depends on the k-solution and on the reduced frequency σ as a parameter. The ζ-solution is valid for all values of σ and 'amplitude', a. Another feature worth noting is that the unsteady part of the solution is independent of position x. Consequently, the unsteady part of the solution can be effectively decoupled from the steady solution.

To solve the boundary value problems (31) and (32) the differential equations were written as a system of first order differential equations. An approximate solution to the nonlinear k-problem was obtained by Newton's method. Both differential equations were approximated by the Centered-Euler method. The solutions obtained are second order accurate. Figure (4) presents k, k' and k'' as a function of η. Figures (5) - (8) present the real and imaginary points of ζ and ζ' as a function of η for select values of reduced frequency, σ.

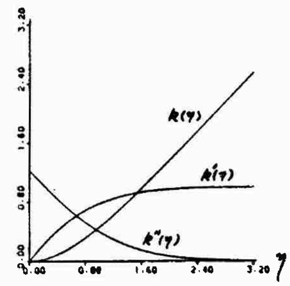

Fig. 4-Steady Flow Functions

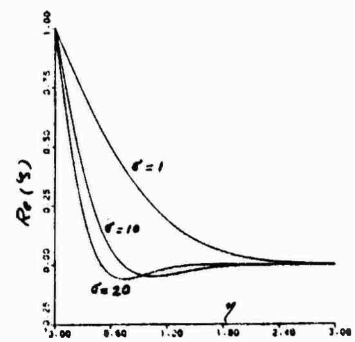

Fig. 5-Variation of Re(ζ) with η

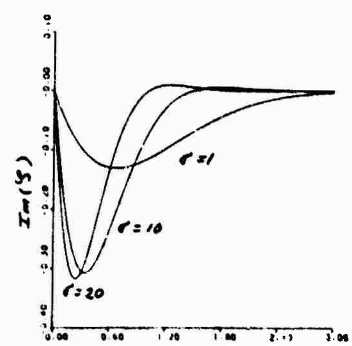

Fig. 6-Variation of Im(ζ) with η

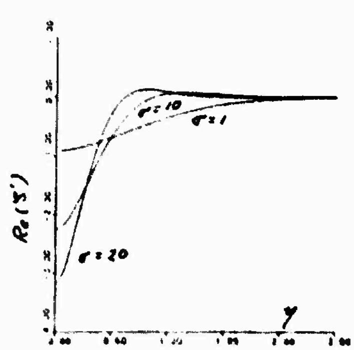

Fig. 7-Variation of Re(ζ') with η

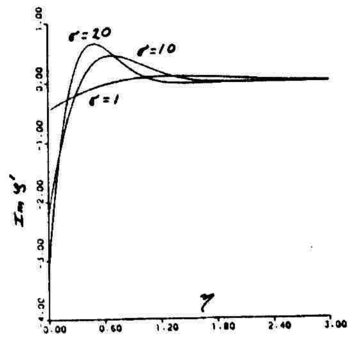

Fig. 8-Variation of Im(ζ') with η

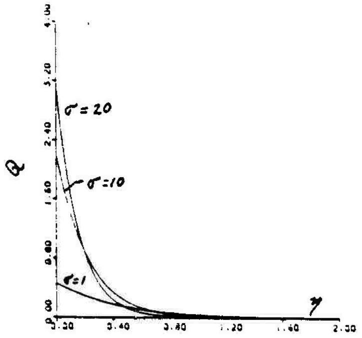

Fig. 9-Variation of Q with η

LEADING EDGE SEPARATION

According to Moore-Rott-Sears[10,11] the point of separation of a boundary layer adjacent to a moving surface occurs when the velocity and the shear stress simultaneously vanish. That is, when $u = u_y = 0$. When the M-R-S conditions are applied to Eq. (30) at the airfoil nose, $x = 0$ and the time dependence is eliminated from the resulting expressions, we obtain

$$Q(\eta;\sigma) \equiv \zeta_R\zeta_I' - \zeta_R'\zeta_I = 0 \tag{33}$$

Here, Q is a function of η that depends parametrically on the reduced frequency, σ.

For any specified value of reduced frequency, if an η-root can be found to the equation $Q(\eta;\sigma) = 0$, then dynamic separation at the airfoil nose, $x = 0$, is believed to occur. Equation (33) has been plotted for a range of σ. The indication is that dynamic stall does not occur at the airfoil nose for any value of σ according to this criterion. See Figure (9). This is not surprising since the $x = 0$ case is strictly a shear wave and symmetry rules out both u and u_y simultaneously vanishing except at the edge of the boundary layer, $\eta \to \infty$.

Applying the M-R-S conditions when $x \neq 0$ gives

$$\left.\begin{aligned} 2Uxf'f'' &= -f''\beta_1\cos\omega t + f''\beta_2\sin\omega t \\ 2Uxf'f'' &= -f'\beta_1'\cos\omega t + f'\beta_2'\sin\omega t \end{aligned}\right\} \tag{34}$$

where

$$\beta_1 = a_R\zeta_R - a_I\,\zeta_I$$

$$\beta_2 = a_R\zeta_I + a_I\zeta_R$$

$$\beta_1' = d\beta_1/d\eta, \quad \beta_2' = d\beta_2/d\eta$$

Eliminating x from Eqs. (34) gives

$$(a_R\alpha_1 + a_I\alpha_2)\cos\omega t = (a_I\alpha_1 - a_R\alpha_2)\sin\omega t \tag{35}$$

where

$$\alpha_1 = \zeta_R f'' - \zeta_R' f'$$

$$\alpha_2 = \zeta_I' f' - \zeta_I f''$$

This equation implies that for $x \neq 0$ and for any values of σ and η, a time t can be found such that the M-R-S conditions are met for the specified harmonic motion. The implication is that the M-R-S conditions do not provide a useful separation criterion for this problem.

We propose the following methodology for predicting separation for this problem:

(i) Set the steady part of u equal to the amplitude of the unsteady part. That is,

$$2Uxk'(\eta) = |a||\zeta| \tag{36}$$

(ii) Set the steady part of the shear stress (which is proportional to u_y) equal to the amplitude of the unsteady part of u_y. That is,

$$2Uxk''(\eta) = |a||\zeta'| \tag{37}$$

Eliminating x from Eqs. (36), (37) gives the leading edge dynamic stall condition. That is,

$$k''/k' = \left[\frac{(\zeta_R')^2 + (\zeta_I')^2}{\zeta_R^2 + \zeta_I^2}\right]^{1/2} \tag{38}$$

The right hand side of Eq. (38) depends parametrically on the reduced frequency, σ. For any specified value of σ an η-value can be found satisfying this equation. Figure (10) presents a graph of the η-root of Eq. (38) as a function of σ.

(iii) The x-location of the separation point is obtained from the expression

$$x = |a||\zeta(\eta_{rt})|/2Uk'(\eta_{rt}) \tag{39}$$

Notice that the amplitude $|a|$ of the unsteady motion comes into the x-location of the separation flow.

(iv) Separation occurs for the motion when the value of x obtained from Eq. (39) is less than or equal to r, where r is the radius of curvature of the airfoil's leading edge. That is, when

$$|x| \leq r \longrightarrow \text{SEPARATION}$$

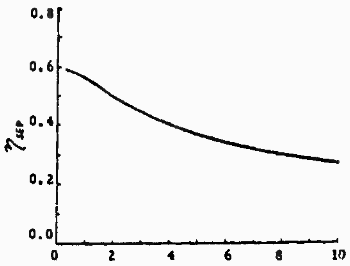

Fig.10-Separation η Variation with σ

This last part of the procedure has been included since the local boundary layer flow is not a valid representation of the flow adjacent to the entire airfoil but only in a region of the order of the radius of curvature of the airfoil nose.

CONCLUDING REMARKS

A methodology has been proposed for predicting leading edge separation due to the accelerating flow about the well-rounded nose of an airfoil. The fluid accelerations are caused by the curvature of the leading edge geometry and the forced transverse oscillations of the airfoil. The analyses leading to an unsteady separation criterion couples the gross features of the airfoil's transverse motions with the details surrounding a laminar boundary layer in the vicinity of its dividing streamline.

The separation criterion has not been validated by comparison with any experimental data. This clearly remains before the procedure can be seriously advanced as a useful tool for predicting leading edge dynamic stall.

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